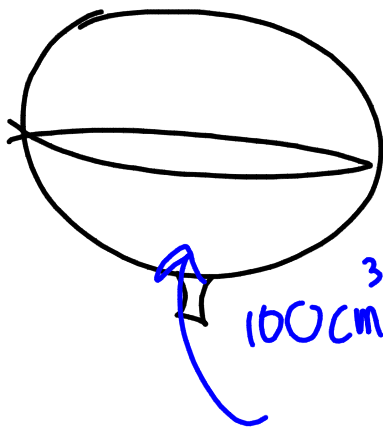


4-6 day 1 Related Rates

Learning Objectives:

I can use derivatives and the process of related rates to find rates in real world situations where I know another rate.

Ex1. Air is being pumped into a spherical balloon so that its volume increases at a rate of $100 \text{ cm}^3/\text{sec}$. How fast is the radius of the balloon increasing when the diameter is 50cm ? $r=25\text{cm}$



$$\frac{dV}{dt} = 100 \text{ cm}^3/\text{sec}$$

$$\frac{dr}{dt} = ?$$

V of Sphere

$$V = \frac{4}{3} \pi r^3$$

$$V(t) = \frac{4}{3} \pi [r(t)]^3$$

$$\frac{dV}{dt} = 4\pi [r(t)]^2 \cdot \frac{dr}{dt}$$

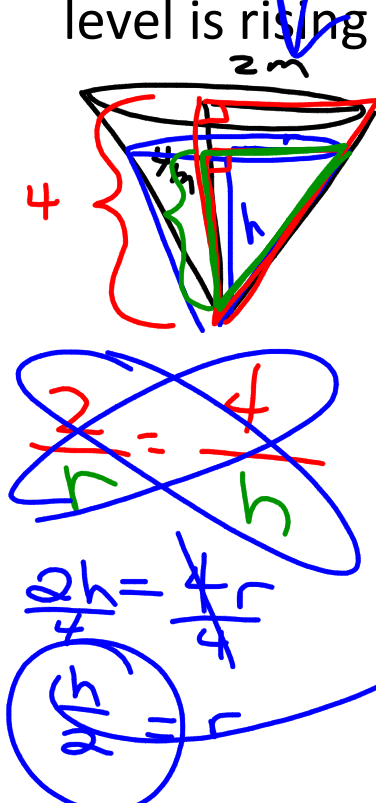
$$100 = 4\pi [25]^2 \cdot \frac{dr}{dt}$$

$$100 = 2500\pi \cdot \frac{dr}{dt}$$

$$\frac{100}{2500\pi} = \frac{dr}{dt}$$

$$\frac{1}{25\pi} \text{ cm/sec} = \frac{dr}{dt}$$

Ex2. A water tank has the shape of an inverted cone with a base radius of 2 m and a height of 4m. If water is being pumped into the tank at a rate of $2 \text{ m}^3/\text{min}$, find the rate at which the water level is rising when the water is 3m deep.



$2 \text{ m}^3/\text{min}$ $\frac{dV}{dt} = 2$ $\frac{dh}{dt} = ?$

Volume of Cone

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{h}{2}\right)^2 h$$

$$V = \frac{1}{3} \pi \frac{h^2}{4} \cdot h$$

$$V = \frac{1}{12} \pi h^3$$

$$V(t) = \frac{1}{12} \pi [h(t)]^3$$

$$\frac{dV}{dt} = \frac{1}{4} \pi h^2 \frac{dh}{dt}$$

$$2 = \frac{1}{4} \pi 3^2 \frac{dh}{dt}$$

$$2 = \frac{9}{4} \pi \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{8}{9\pi} \text{ m/min}$$

Homework

pg 251 # 11, 13, 15, 16, 17, 19, 20,
22, 27, 33, 35, 38, 42